

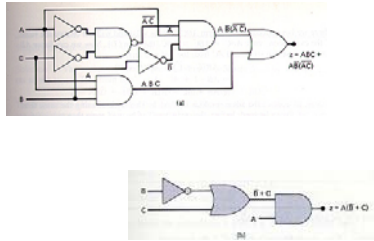
NAND gate only


$$(17) \quad \overline{(x \cdot y)} = \bar{x} + \bar{y}$$

COMBINATORIAL LOGIC CIRCUITS

ALGEBRAIC SIMPLIFICATION

Example 1



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EXAMPLE

EXAMPLE 4-3

Simplify $z = \bar{A}C(\bar{A}BD) + \bar{A}BC\bar{D} + \bar{A}BC$.

Solution First, use DeMorgan's theorem on the first term:

$$z = \bar{A}C(A + \bar{B} + \bar{D}) + \bar{A}BC\bar{D} + \bar{A}BC \quad (\text{step 1})$$

Multiplying out,

$$z = \bar{A}CA + \bar{A}C\bar{B} + \bar{A}C\bar{D} + \bar{A}BC\bar{D} + \bar{A}BC \quad (2)$$

Since $\bar{A} \cdot A = 0$, the first term is eliminated:

$$z = \bar{A}C\bar{B} + \bar{A}C\bar{D} + \bar{A}BC\bar{D} + \bar{A}BC \quad (3)$$

This is the desired sum-of-products form. Now we have to look for common factors among the various product terms. The idea is to check for the largest common factor between any two or more product terms. For example, the first and last terms have the common factor $\bar{A}C$, and the second and third terms share the common factor $\bar{A}B$. We can factor these out as follows:

$$z = \bar{A}C(\bar{B} + B) + \bar{A}B(\bar{C} + \bar{D}) \quad (4)$$

Now, since $\bar{B} + B = 1$, and $C + \bar{C} = C + B$ [theorem (15)], we have

$$z = \bar{A}C + \bar{A}B(C + B) \quad (5)$$

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EXAMPLE

EXAMPLE 4-4

Simplify the expression $x = (\bar{A} + B)(A + B + D)\bar{D}$.

Solution The expression can be put into sum-of-products form by multiplying out all the terms. The result is

$$x = \bar{A}A\bar{D} + \bar{A}B\bar{D} + \bar{A}D\bar{D} + BA\bar{D} + BB\bar{D} + BD\bar{D}$$

The first term can be eliminated, since $\bar{A}A = 0$. Likewise, the third and sixth terms can be eliminated, since $D\bar{D} = 0$. The fifth term can be simplified to $B\bar{D}$, since $BB = B$. This gives us

$$x = \bar{A}B\bar{D} + AB\bar{D} + B\bar{D}$$

We can factor $B\bar{D}$ from each term to obtain

$$x = B\bar{D}(\bar{A} + A + 1)$$

Clearly, the term inside the parentheses is always 1, and so we finally have

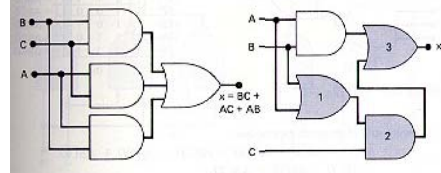
$$x = B\bar{D}$$

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EXAMPLE

A	B	C	x
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a)



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EXAMPLE

Topdown design

1 Block diagram

2 Truth table

3 Boolean function

4 S the following sum-of-products expression:

$$z = \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}BCD + \bar{A}BC\bar{D}$$

$$+ \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}BCD$$

5 S Simplification of this expression will be a formidable task, but with a little care it can be accomplished. The step-by-step process involves factoring and eliminating terms of the form $A + \bar{A}$:

$$z = \bar{A}BCD + \bar{A}BC(\bar{D} + D) + \bar{A}BC(\bar{D} + D) + \bar{A}BC(\bar{D} + D) + \bar{A}BC(\bar{D} + D)$$

$$= \bar{A}BCD + \bar{A}BC + \bar{A}BC + \bar{A}BC + \bar{A}BC$$

$$= \bar{A}BCD + \bar{A}B(\bar{C} + C) + \bar{A}B(\bar{C} + C)$$

$$= \bar{A}BCD + \bar{A}B + \bar{A}B$$

$$= \bar{A}BCD + \bar{A}B + \bar{A}B$$

$$= \bar{A}BCD + \bar{A}B + \bar{A}B$$

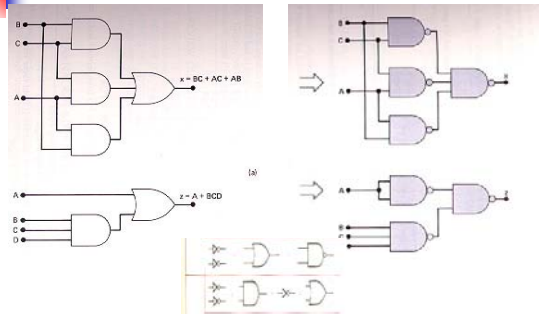
$$= \bar{A}BCD + \bar{A}B + \bar{A}B$$

This can be reduced further by invoking theorem (15), which says that $x + \bar{x}y = x + y$. In this case $x = \bar{A}B$ and $y = \bar{A}BCD$. Thus,

$$z = \bar{A}BCD + \bar{A}B + \bar{A}BCD + \bar{A}B$$

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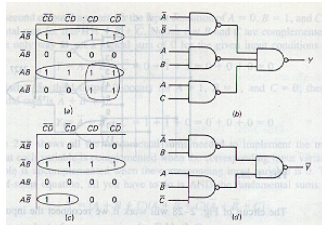
EXAMPLE Converting circuits to NAND.



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Example

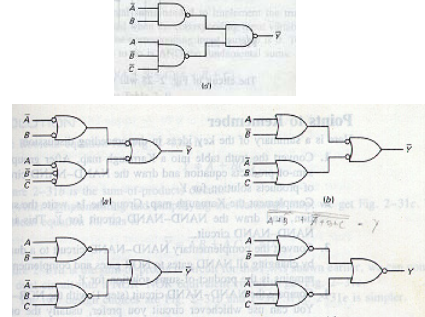
- Sum-of-products NAND-NAND circuit



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Example

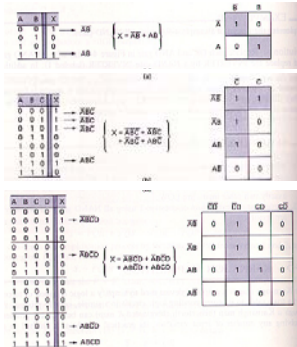
- products -of- Sum NOR-NOR circuit



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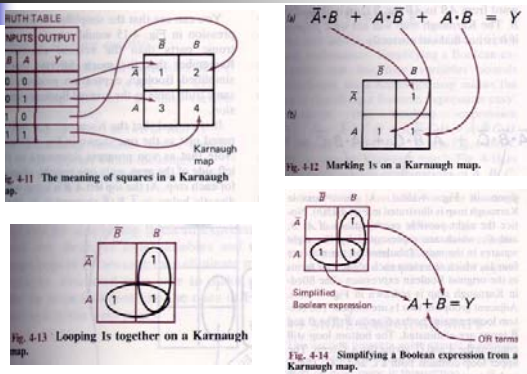
KANAUGH MAP METHOD

KANAUGH MAP AND TRUTH TABLE

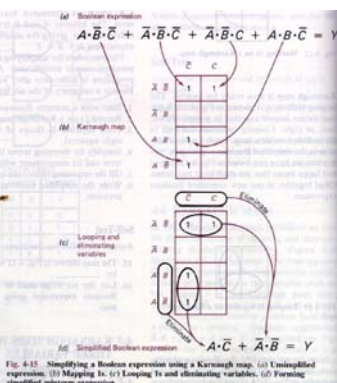


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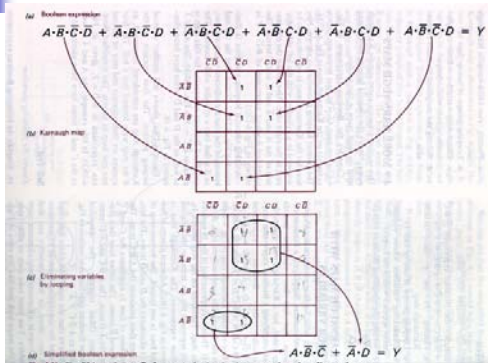
KANAUGH MAP



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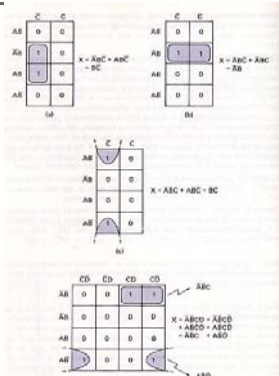


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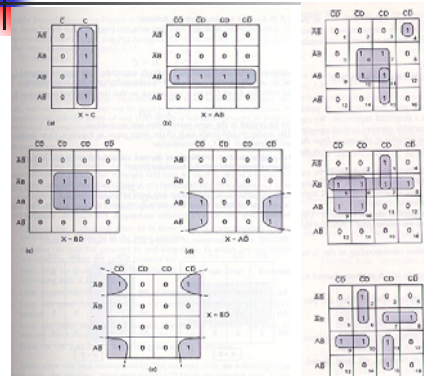
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Example of looping pairs



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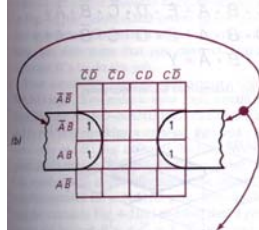
Example of looping groups of four



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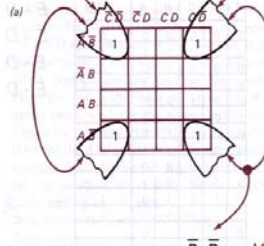
(a) Boolean expression

$$A \cdot B \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot B \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot B \cdot C \cdot \bar{D} + A \cdot B \cdot C \cdot \bar{D} = Y$$



(b) Simplified Boolean expression $B \cdot \bar{D} = Y$

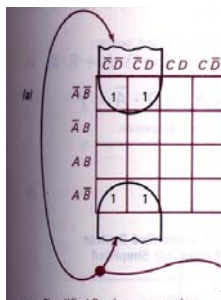
Fig. 4-17 Simplifying a Boolean expression using a Karnaugh map. By considering the map as a vertical cylinder, the four 1s can be looped as shown.



(b) Simplified Boolean expression $\bar{B} \cdot \bar{D} = Y$

Fig. 4-19 Simplifying a Boolean expression by thinking of the Karnaugh map as a ball. In this way, the 1s in the four corners can be looped.

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(b) Simplified Boolean expression $\bar{B} \cdot \bar{C} = Y$

Fig. 4-18 Simplifying a Boolean expression by considering the Karnaugh map as a horizontal cylinder. In this way, the four 1s can be looped.

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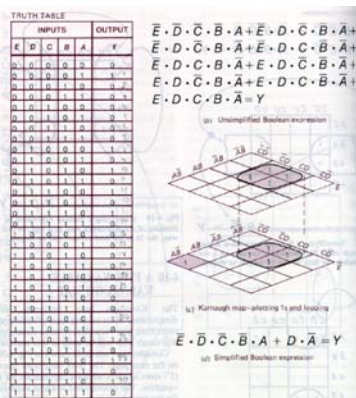


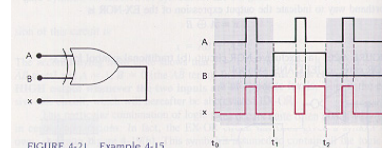
Fig. 4-20 A logic problem with five variables. (a) Truth table. (b) Unimplified Boolean expression. (c) Plotting 1s and looping on a five-variable Karnaugh map. (d) Simplified Boolean expression.

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EXCLUSIVE-OR & EXCLUSIVE-NOR CIRCUITS

EXAMPLE 4-15

Determine the output waveform for the input waveforms given in Figure 4-21.

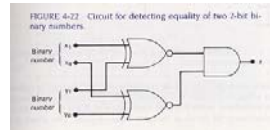


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EXCLUSIVE-OR & EXCLUSIVE-NOR CIRCUITS

- Design a logic circuit, using x_1, x_0, y_1, y_0 inputs, whose output will be HIGH only when the two binary numbers x_1x_0 and y_1y_0 are *equal*.

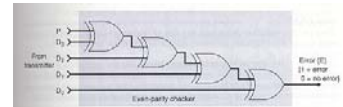
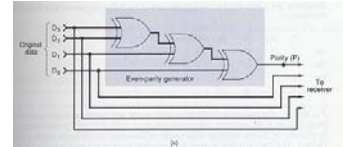
x_1	x_0	y_1	y_0	x (OUTPUT)
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1



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PARITY GENERATOR AND CHECKER

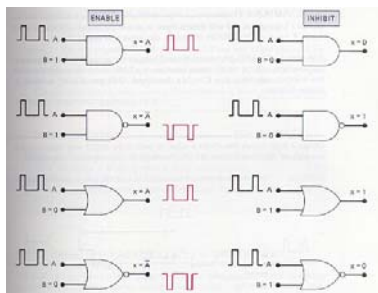
- EX-OR gates used to implement parity generator and parity checker for even parity system.



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INHIBIT CIRCUITS

- Four basic gates can either enable or inhibit the passage of and input signal, A , under control of the logic level at control input B .



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- $F(A,B,C,D,E) = \sum_m (2,3,7,10,11,15,18,19,23,24,25,26,27,28,29,30,31)$

- $F(A,B,C,D,E) = DE + AB + \bar{C}D$

$\bar{A}$ BC	00	01	11	10
DE 00	0	4	12	8
DE 01	1	5	13	9
DE 11	3	7	15	11
DE 10	2	6	14	10

A BC	00	01	11	10
DE 00	16	20	28	24
DE 01	17	21	29	25
DE 11	19	23	31	27
DE 10	18	22	30	26

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- $F(A,B,C,D,E,F) = \sum_m (0,1,4,5,6,7,8,9,10,11,12,13,20,21,22,23,26,27,32,42,43,47,48,58,59,61)$

$\bar{A} \bar{B}$ CD	00	01	11	10
EF 00				
EF 01				
EF 11				
EF 10				

AB CD	00	01	11	10
EF 00				
EF 01				
EF 11				
EF 10				

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- $F(A,B,C,D,E,F) = \sum_m (0,1,4,5,6,7,8,9,10,11,12,13,20,21,22,23,26,27,32,42,43,47,48,58,59,61)$

$\bar{A} \bar{B}$ CD	00	01	11	10
EF 00	0	4	12	8
EF 01	1	5	13	9
EF 11	3	7	15	11
EF 10	2	6	14	10

AB CD	00	01	11	10
EF 00	16	20	28	24
EF 01	17	21	29	25
EF 11	19	23	31	27
EF 10	18	22	30	26

$$\bar{A}BE + \bar{A}CD + CDE + ACDEF + \bar{A}BCEF + ABCDEF$$

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- $F(A,B,C,D,E) = \Sigma_m (1,3,12,13,14,16,17,18, 19)$
- $F(A,B,C,D,E) = \Sigma_m (5,8,10,11,12,13,24,28,29)$